CS 0368-4246: Combinatorial Methods in Algorithms (Spring 2025) March 31, 2025

Lecture 3: Even Cycle Finding and Fine-Grained Complexity

Instructor:  $Or \ Zamir$ 

## **Homework Questions**

- 1. Consider a Data Structure that maintains a **directed** graph G on n vertices and supports the following operations:
  - Init(n) initializes an empty directed graph on n vertices V = [n].
  - AddEdge(u, v) adds a directed edge between two vertices  $u, v \in [n]$ .
  - DeleteEdge(u, v) removes the directed edge (u, v) from the graph.
  - Reach(u) returns the number of vertices  $v \in [n]$  such that v is *reachable* from u in G (i.e., there exists a directed path from u to v).

We denote by m the number of edges stored in the structure.

- (a) Show that all operations can be implemented in O(m) time per operation (worst case).
- (b) Assuming SETH, show that there is no implementation of all operations in  $m^{1-\varepsilon}$  amortized time, for any constant  $\varepsilon > 0$ .